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AdS/CFT DUALITY AND CONFORMALITY FOR NON-ABELIAN ORBIFOLD

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An outline of the conformality approach to the gauge hierarchy is given including the use of non-abelian orbifolds to give unified models of the left-right type.

One can identify four approaches to the hierarchy problem by different new (untested) physics at a TeV scale:

- Supersymmetry. GUT unification works better with TeV supersymmetry.
- Technicolor. New strong dynamics at a TeV scale.
- Large extra dimensions at a TeV scale.
- Conformality at a TeV scale.

Here, I discuss the fourth approach. It is motivated physically by the notion that at a TeV scale the standard model appears almost conformally invariant in the sense that the masses of the particles, as well as the QCD and weak scales appear almost vanishingly small. But the theory is *not* conformal invariant as it stands because the couplings still run. The conformality idea is to enrich the spectrum just so that the couplings cease to run at an infra-red fixed point of the renormalization group.

Nevertheless as will become clear there can still be a gauge coupling unification at the TeV scale in a larger gauge group. The disparity of the 321 couplings then arises from the group theory of the embedding in the larger group.

Using AdS/CFT duality, one arrives at a class of gauge field theories of special recent interest. The simplest compactification of a ten-dimensional superstring on a product of an AdS space with a five-dimensional spherical manifold leads to an $\mathcal{N} = 4$ $SU(N)$ supersymmetric gauge theory, well-known to be conformally invariant[1]. By replacing the manifold S^5 by an orbifold S^5/Γ one arrives at less supersymmetries corresponding to $\mathcal{N} = 2,\ 1$ or 0 depending on whether $\Gamma \subset SU(2),\ \ SU(3),\ $ or $\not\subset SU(3)$ respectively, where Γ is in all cases a subgroup of $SU(4) \sim SO(6)$ the isometry of the S^5 manifold.

It was conjectured in [2] that such $SU(N)$ gauge theories are conformal in the $N \to \infty$ limit. In [3] it was conjectured that at least a subset of the resultant

nonsupersymmetric $\mathcal{N} = 0$ theories are conformal even for finite N. Some first steps to check this idea were made in [4]. Model-building based on abelian Γ was studied further in [5,6,7], arriving in [7] at an $SU(3)^7$ model based on $\Gamma = Z_7$ which has three families of chiral fermions, a correct value for $\sin^2\theta$ and a conformal scale ~ 10 TeV.

The case of non-abelian orbifolds bases on non-abelian Γ has now been studied[8]. We have considered all non-abelian discrete groups of order $g < 32$. These are described in detail in [9,10]. There are exactly 45 such non-abelian groups. Because the gauge group arrived at by this construction[5] is $\otimes_i SU(Nd_i)$ where d_i are the dimensions of the irreducible representations of Γ, one can expect to arrive at models such as the Pati-Salam $SU(4) \times SU(2) \times SU(2)$ type[11] by choosing $N = 2$ and combining two singlets and a doublet in the **4** of $SU(4)$. Indeed we find that such an accommodation of the standard model is possible by using a non-abelian Γ.

The procedures for building a model within such a conformality approach are: (1) Choose Γ; (2) Choose a proper embedding $\Gamma \subset SU(4)$ by assigning the components of the **4** of $SU(4)$ to irreps of Γ, while at the same time ensuring that the **6** of $SU(4)$ is real; (3) Choose N, in the gauge group $\otimes_i SU(Nd_i)$.

We choose $N = 2$ and aim at the gauge group $SU(4)\times SU(2)\times SU(2)$. To obtain chiral fermions, it is necessary[5] that the **4** of $SU(4)$ be complex $\mathbf{4} \neq \mathbf{4}^*$. Actually this condition is not quite sufficient to ensure chirality in the present case because of the pseudoreality of $SU(2)$. We must ensure that the **4** is not pseudoreal.

This last condition means that many of our 45 candidates for Γ do not lead to chiral fermions. For example, $\Gamma = Q_{2n} \subset SU(2)$ has irreps of appropriate dimensionalities for our purpose but it will not sustain chiral fermions under $SU(4)\times SU(2) \times SU(2)$ because these irreps are all, like $SU(2)$, pseudoreal.* Applying the rule that **4** must be neither real nor pseudoreal leaves a total of only 19 possible non-abelian discrete groups of order $g \leq 31$. The smallest group which avoids pseudoreality has order $g = 16$ but gives only two families. The technical details of our systematic search iare in [8].

Here we mention only the simplest interesting non-abelian case which has $g = 24$ and gives three chiral families in a Pati-Salam-type model[11].

The first group that can lead to exactly three families occurs at order $g = 24$ and is $\Gamma = Z_3 \times Q$ where $Q(\equiv Q_4)$ is the group of unit quarternions which is the smallest dicyclic group Q_{2n}.

There are several potential models due to the different choices for the **4** of $SU(4)$ but only the case $\mathbf{4} = (1\alpha, 1^{'}, 2\alpha)$ leads to three families.

Since $Q \times Z_3$ is a direct product group, we can write the irreps as $r_i \otimes \alpha^a$ where r_i is a Q irrep and α^a is a Z_3 irrep. We write Q irreps as $1,\ 1^{'},\ 1^{''},\ 1^{'''},\ 2$ while the irreps of Z_3 are all singlets which we call $1,\ \alpha, |\alpha^{-1}$. Thus $Q \times Z_3$ has twelve irreps in all and the gauge group will be of Pati-Salam type for $N = 2$.

If we wish to break all supersymmetry, the **4** may not contain a singlet of Γ. Due to permutational symmetry among the singlets it is sufficiently general to choose **4**

*Note that were we using $N \geq 3$ then a pseudoreal **4** would give chiral fermions.

$= (1\alpha^{a_1},\ 1^{'}\alpha^{a_2},\ 2\alpha^{a_3})$ with $a_1 \neq 0$.

To fix the a_i we note that the scalar sector of the theory which is generated by the **6** of $SU(4)$ can be used as a constraint since the **6** is required to be real. This leads to $a_1 + a_2 = -2a_3 (\text{mod } 3)$. Up to permutaions in the chiral fermion sector the most general choice is $a_1 = a_3 = +1$ and $a_2 = 0$. Hence our choice of embedding is

$$\mathbf{4} = (1\alpha,\ 1^{'},\ 2\alpha) \tag{1}$$

with

$$\mathbf{6} = (1^{'}\alpha,\ 2\alpha,\ 2\alpha^{-1},\ 1^{'}\alpha^{-1}) \tag{2}$$

which is real as required.

We are now in a position to summarize the particle content of the theory. The fermions are given by

$$\sum_I \mathbf{4} \times R_I \tag{3}$$

where the R_I are all the irreps of $\Gamma = Q \times Z_3$.

The scalars are given by

$$\sum_I \mathbf{6} \times R_I \tag{4}$$

As described in more detail in [8] the scalars are suffcient to break the starting gauge symmetry $SU(4)^3 \times SU(2)^{12}$ to the required 4-2-2 left-right structure, and with precisely three chiral families in 16-plets.

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